

Research on Asset Pricing Model with Double Delays Based on Heterogeneous Beliefs

Zhang Handan

School of Economics and Management, Southwest University, Chongqing, China, 400715

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Abstract: In the classical asset pricing theory, the individual investors are usually selected as analysis point. However, in the modern financial market, a large amount of wealth is not managed by investors themselves, but professional institutions. In terms of traditional behavioral asset pricing model, a single representative individual is used to describe all the agency investment institutions, ignoring the competitive behaviors among institutional investors that care about relative performance. Based on this consideration, an asset pricing model with double time delays based on investors' heterogeneous beliefs is established this paper. The existence of heterogeneous beliefs will affect the stock price under the market equilibrium state and make it deviate from the real value, which will also push up the volatility of stocks, make the market prices of risks identified by agents different and show anti-economic cycle characteristics. Based on the trading data of individual stocks in China, this paper empirically analyzes the short-term impact of heterogeneous beliefs on stock prices, and verifies that the higher the heterogeneous beliefs of investors, the higher the weekly return rate, while the highest heterogeneous beliefs will show a significant reversal next week.

1. Introduction

Research on asset pricing has achieved rich theoretical results in the 1970s, mainly including modern portfolio theory, capital asset pricing model, arbitrage pricing theory and option pricing theory [1]. Under the basic premise of "rational man hypothesis" and "efficient market hypothesis", these theories focus on studying about how people reasonably allocate scarce resources across periods and giving an internal logical unified explanation to the price phenomenon in financial markets. At the same time, more and more empirical studies have found various financial anomalies inconsistent with the results of classical financial theories, such as long-standing "month effect puzzle", "equity premium puzzle", "overreaction" and "underreaction" phenomena [2], which are contrary to the results of classical theories and have been the focus of financial research since their discovery. In order to seek a more perfect explanation of the market, some scholars try to introduce more general theoretical assumptions to expand the traditional asset pricing model. Since Kahneman and Tversky (1979) put forward the famous prospect theory, behavioral finance research has developed rapidly. Scholars in the field of asset pricing began to analyze bounded rational investors in order to seek new theoretical breakthroughs. The behavioral asset pricing theory developed from this successfully explains a large number of financial anomalies in asset pricing by introducing individual systematic cognitive biases into the model.

The traditional asset pricing model uses a representative agent to describe the behavior of all investors under the assumption of a complete market, and the agent's judgment on information is completely consistent with the economic environment. Due to the existence of representative individuals, the dynamic interaction between different investors and its influence on financial markets have been neglected [3]. An important research direction of behavioral asset pricing theory is to establish the model in the economic environment of heterogeneous individuals, and analyze the influence of investors' interactive investment behavior on asset prices. The early noise trading model and the dynamic asset pricing model established under the pure exchange economy in recent years have made theoretical breakthroughs through the behavior characteristics of heterogeneous investors. Heterogeneous belief is a common assumption for heterogeneous individuals in

behavioral asset pricing theory models [4]. Investors have different opinions on the same asset either because of different information received or because of prior heterogeneity. Theoretical and empirical studies based on heterogeneous beliefs have successfully explained the formation mechanism of asset bubbles, momentum effect and reversal effect, etc., which are common in the market, and have greatly promoted the development of asset pricing research.

From the end of 2014 to the middle of 2015, China's share market experienced a spectacular "mad cow" market and a plummeting stampede "stock market crash". The market experienced ups and downs and never formed a reasonable pricing mechanism. Miller (1977) analyzed the mechanism of asset bubble formation in the presence of heterogeneous beliefs and short selling restrictions, who believes that the existence of short selling restrictions in the real financial market makes the information of pessimistic investors unable to be fully expressed through free trade, resulting in the stock price being overvalued because it mainly contains information of optimistic investors [5]. The greater the difference in beliefs between optimistic and pessimistic investors, the more seriously the stock price is overvalued, and there has been strict short selling restrictions in China's securities market for more than 20 years since its establishment.

Based on the above theoretical and practical background, this paper integrates heterogeneous beliefs and institutional investors' consideration of relative performance into asset pricing research, and establishes an asset pricing model considering investors' heterogeneous beliefs and relative performance under the framework of Nash equilibrium. The main purpose of the study is to discuss the investment behavior of institutional investors based on the consideration of relative performance and its impact on asset pricing from both theoretical model and actual situation. The introduction of heterogeneous beliefs, on the one hand, makes the two types of institutional investors in the model have different expectations of the market and have interactive investment behaviors; on the other hand, it explores the pricing of behavioral assets under the joint influence of various behavioral factors. This paper hopes to uphold the theoretical results related to heterogeneous beliefs, give a reasonable explanation to the behavior of institutional investors, especially the interactive investment behavior based on relative performance, and study related asset pricing issues.

2. Heterogeneous Beliefs

The traditional asset pricing model uses an agent to describe the investment behavior of all investors under the assumption of a complete market, and the agent's judgment on information is completely consistent with the economic environment. In such an economic environment, the agent's beliefs and consumption preferences determine the relationship between the overall economy and asset prices. However, due to the existence of representative individuals, the interaction between investors has been neglected, which makes the model unable to describe investment phenomena such as herding behavior and game behavior, and also unable to give a perfect explanation of asset price behavior in the actual market. In the face of financial anomalies found in empirical studies, scholars have begun to consider introducing individuals with heterogeneity into pricing models, in order to further study the pricing mechanism of assets.

Heterogeneous belief is one of the common hypotheses of heterogeneous individual economy. Different from the assumption in the traditional asset pricing model that all investors have the same judgment on the economic environment, heterogeneous beliefs refer to the differences in investors' expectations of the same asset. When all investors in the market can get all the information free of charge and treat the information in exactly the same way, their expectations of assets can be consistent, and the homogeneous belief hypothesis of the unified asset pricing model can be established. However, in the real financial market, the above premise obviously cannot be satisfied. All investors cannot obtain information at the same time, and different investors have different understanding of information. Therefore, for investors' beliefs, the assumption that different investors in the same market have different expectations for the return rate of the same asset is more realistic.

Hongandstein (2007) discussed three reasons for investors to disagree, namely, the gradual flow of information, limited attention and transcendental heterogeneity. These three theories emphasize

different angles, but they have something in common [6]. The smooth flow of information means that all investors cannot obtain information at the same time due to the technical channels of information diffusion and the different specialities of investors, which will cause some investors to update their judgment on stocks after receiving the new information, and have different beliefs with investors who have not received the information (Hong and Stein, 1999). At the same time, behavioral finance studies have found that investors often believe too much in the information they get and do not realize their disadvantages in information acquisition, which further prevents them from learning the investment behavior of other investors quickly, and makes the difference in beliefs sustainable. The limited attention of investors refers to a situation similar to the smooth flow of information, but no longer emphasizes the dynamic dissemination of information, but emphasizes the degree of attention of investors to specific information. However, even if investors can obtain and fully notice all the public information at the same time, their different prior knowledge of the economic environment will lead to the emergence of heterogeneous beliefs.

3. An Empirical Study on the Influence of Heterogeneous Beliefs on Stock Prices

Due to the lack of visible data reflecting heterogeneous beliefs, this paper uses market sentiment variables reflecting the overall market situation to construct heterogeneous beliefs. Most of the previous empirical studies on market sentiment have only constructed market sentiment with a single frequency, without paying attention to the difference in the influence of different frequency sentiment indicators on stock returns. McClure, Aibson and Loewenstein (2004) have studied the neural mechanism of time discount and come to the conclusion that irrational factors will have a greater effect on short-term decision-making [7]. Based on this experimental result, this paper uses the annualization algorithm to examine the influence of market sentiment of different frequencies, and discusses whether the influence of short-term sentiment on stock index returns exceeds that of long-term sentiment. Traditional measurement methods will lose some high-frequency information. In this paper, according to mixing data extraction model (MIDAS) proposed by Ghysels, Santa-Clara and Valkanov (2004), mixed market emotions of different frequencies are processed to obtain mixed market emotions, and the linkage between mixed market emotions and stock index returns is studied.

3.1 Indicator Selection

Baker (2002) [5] thinks that the market transaction volume can well reflect the high and low market sentiment. When market sentiment is high, investors' enthusiasm for investment will be very high, and the volume of market transactions will be large, while vice versa. In addition, the higher the market sentiment, the more active the market transactions, the higher the stock turnover rate. Therefore, the turnover rate can also be used as an emotional proxy indicator. Based on the domestic and foreign literatures and the actual market conditions in China, this paper selects Shanghai Stock Exchange Volume (SZV), Closed-end Fund Discount RATE (CFD), Turnover Rate (TURN), Rise and Fall Ratio (ARMS) of Rising Stock and Falling Stock, and Shanghai Stock Exchange Rate in the previous period as proxy variables of current mood.

In this study, rise-fall ratio and return rate data, turnover rate and discount rate data are selected from RESSET database. The selected data period is from October 1, 2010 to September 30, 2015, and the frequency is divided into weeks, months and seasons.

3.2 The Relationship between Market Mood and Stock Price Changes

Among the eigenvalues of the covariance of the final proxy variables, only the eigenvalues of the first component are significantly greater than 1, and the first principal component contribution is 63.92%, which means SI can represent 63.92% of the information of the five variables such as Shanghai Stock Exchange turnover. Eviews software is used to calculate the basic statistical characteristics of market sentiment variables in each data cycle as shown in Table 1. From Table 1, we can find that the skewness of emotional factors deviates from 0, and the kurtosis deviates from 3 besides weekly data. Therefore, we reject the assumption that sentiment index obeys normal

distribution. The changes of market sentiment and Shanghai stock exchange rate of return in week, month and quarter are shown in Fig.1~3 respectively. Although the monthly data deviated in the last part, the weekly data and most of the monthly data showed almost the same trend. Therefore, we can intuitively get the conclusion that the trend of market sentiment index and market index yield is roughly similar from the graph, which shows that the constructed market sentiment index can well reflect the changes of market sentiment.

Tab.1 Statistical Characteristics of Market Emotion

Cycle	Mean Value	Maximum Value	Minimum Value	Standard Deviation	Skewness	Kurtosis
Week	0	8.1823	-3.3005	2.1141	1.845	3.205
Month	0	7.3795	-3.0486	2.3227	1.612	2.258
Season	0	8.0675	-3.8823	3.1962	1.402	1.156

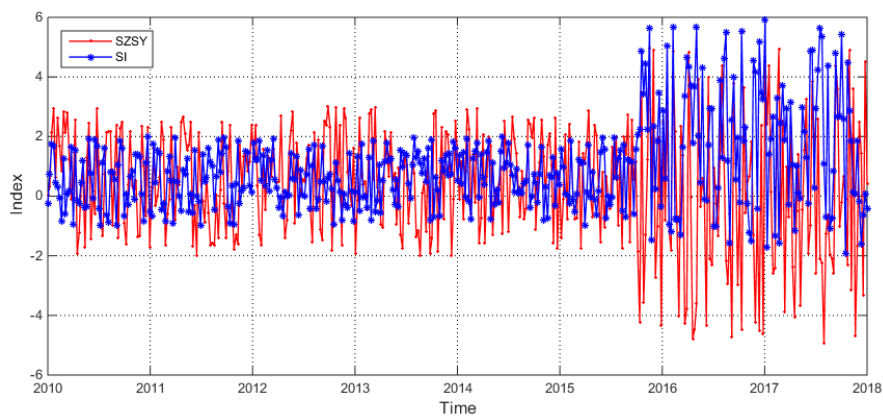


Fig.1 Weekly market sentiment and weekly Shanghai earnings

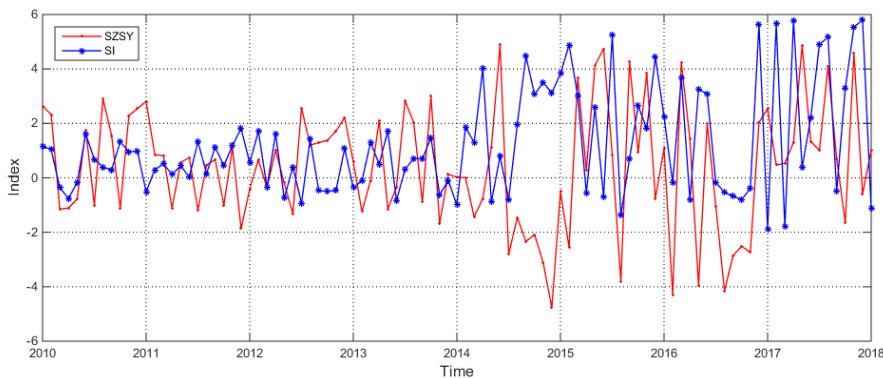


Fig.2 Monthly market sentiment and monthly Shanghai earnings

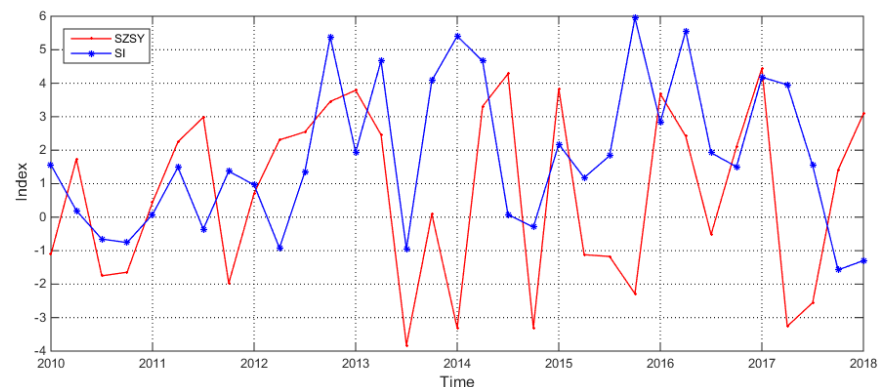


Fig.3 Quarterly market sentiment and quarterly Shanghai earnings

4. Conclusion

The existing behavioral asset pricing studies often only consider the specific behavior patterns of investors to give explanations for specific financial phenomena, and seldom involve investment decision-making and dynamic asset pricing under the joint influence of various behavioral factors. Based on this background, this study attempts to consider the influence of heterogeneous beliefs and relative performance under a unified framework, and specifically analyzes how the heterogeneous beliefs of investors will affect the market equilibrium and how the existence of relative performance will affect the decision-making behavior of institutional investors. Therefore, the work of this paper will complement the research on behavioral asset pricing. At present, the degree of specialization and institutionalization of China's capital market is still at a relatively low level. The market is still dominated by small and medium-sized retail investors, and the stock market often shows a sharp rise and fall. Based on the theory of asset bubble formation under the dual effects of heterogeneous beliefs and short selling restrictions, this paper empirically studies the short-term impact of heterogeneous beliefs on China's stock market. The empirical results show that the introduction of short selling mechanism can weaken the short-term pricing bias caused by investor differences to a certain extent and improve the effectiveness of the market.

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